APPROXIMATE SOLUTION OF A PROBLEM ON THE DYNAMICS OF A BURIED CYLINDRICAL SHELL SUBJECTED TO A LIVE LOAD

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The problem of the action of a live load on underground installations involves the study of the interaction of the structure with its environment and an incident wave. This problem is very complex and is usually solved by numerical methods, although it is interesting that sufficiently rigorous analytic solutions can also be found. The latter are used to check the accuracy of the numerical results, as well as being of independent value.

1. Let a loading wave of length ℓ move over the surface of a half-space filled with soft soil. The wave front has the constant velocity D. We assume that the wave has a triangular form which remains unchanged as it propagates. The pressure function in the wave has the form (Fig. 1)

$$p_{00} = P_0(1 + y)H(Dt_0 - x_0),\ -1 \leqslant y \leqslant 0, \tag{1.1}$$

where $H(Dt_0 - x_0)$ is the unit function; $y = y_0/l$; $y_0 = x_0 - Dt_0$. A hollow, circular thin-walled cylindrical shell of radius R_0 is located in the half-space at the depth H_0. The thickness of the wall of the shell is h. It is necessary to determine the stresses and strains of the shell under the influence of a loading wave generated in the half-space due to the action of live load (1.1).

We will determine the parameters of the wave and the motion of the medium by means of the method in [1], where a solution was found for a quasistatic problem concerning the propagation of three-dimensional waves in an ideal inelastic medium. We assume that the diagrams depicting the bulk deformation of the medium during loading and unloading are linear but different. Instantaneous loading occurs at the front, while unloading of the medium occurs behind the front.

We will examine the case when the velocity D is greater than velocity of propagation of the loading strains in the soil a but less than the velocity of the unloading strains ($a \le D \le a_1$). Then the motion of the medium occurs in a region shaped in the form of a wedge and located between the front of the loading wave and the boundary of the medium. The pressure in the soil in the perturbation region, written in the moving cylindrical coordinate system r_1, α (see Fig. 1) connected with the loading front, is determined from the formula

$$\begin{gathered} p(r_1, \alpha) = p_0\left\{1 + \frac{r_1 \sin(\alpha + \beta + \zeta)}{\sin(\beta + \zeta)} - \right. \\ \left. - \sum_{n=1,2,3,\ldots}^{\infty} \frac{(-1)^n r_1^{\alpha_n} \sin\left[\alpha_n(\alpha - \alpha^+) + \zeta\right]}{\alpha_n[\pi n - (\zeta + \beta)]}\right\},\quad r_1 \leqslant 1, \\ p(r_1, \alpha) = p_0 \sum_{n=1,2,3,\ldots}^{\infty} \frac{(-1)^n r_1^{-\beta_n} \sin\left[\beta_n(\alpha - \alpha^+) - \zeta\right]}{\beta_n[\pi n + (\zeta + \beta)]},\quad r_1 > 1. \end{gathered} \tag{1.2}$$

Here $p_0 = \frac{P_0}{\rho D^2}$; $r_1 = kr$; $k^2 = 1 - \frac{D^2}{a_1^2}$; $\sin\beta = \frac{a}{D}$; $\operatorname{tg}\zeta = \frac{k}{k_1}$; $k_1 = \operatorname{ctg}\beta$; $a^2 = \frac{K_{1d}}{\rho}$; $a_1^2 = \frac{K_{ud}}{\rho}$; $\alpha_n = \frac{1}{\beta}(\pi n - \zeta)$; $\beta_n = \frac{1}{\beta}(\pi n + \zeta)$; ρ is the densityof the medium; α^+ is the angular coordinate of the perturbation

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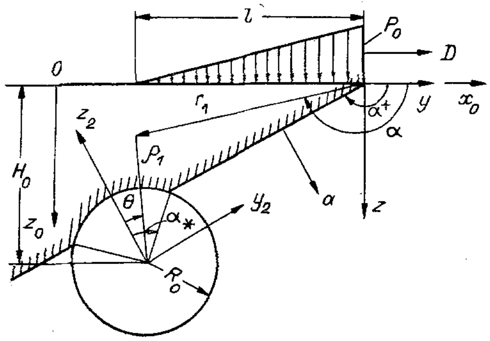

Fig. 1

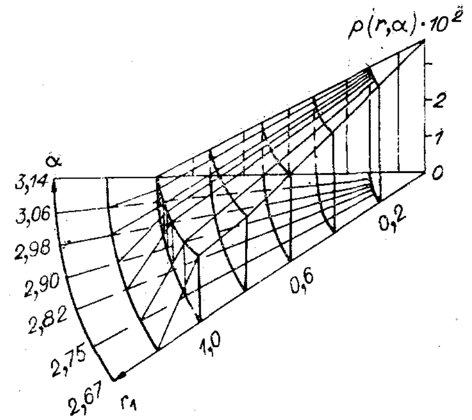

Fig. 2

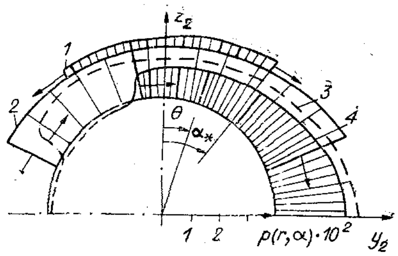

Fig. 3

front in the medium; K_{ld} and K_{ud} are the bulk elastic moduli for the loading and unloading branches of the compression diagram. The coordinates of the polar and moving rectangular coordinate systems are connected by the relations $r^2 = z^2 + y_1^2$, $y_1 = yk^{-1}$, $z = r\sin\alpha$, $y_1 = r\cos\alpha$. In the moving coordinate system, the parameters of the wave and the motion of the medium are independent of the time.

When the velocities coincide (D = a), the angle $\beta = \pi/2$ and

$$p(r_1, \alpha) = p_0\left\{1 + r_1\left[\left(\frac{2\alpha}{\pi} - 1\right)\cos\alpha + \frac{2}{\pi}(\ln r_1 - 1)\sin\alpha\right] - \right.$$
$$\left. - \sum_{n=1,2,3,\ldots}^{\infty} \frac{(-1)^n r_1^{(2n+1)} \cos[(2n+1)(\alpha - \alpha^+)]}{\pi n(2n+1)}\right\}, \quad r_1 \leqslant 1, \qquad (1.3)$$
$$p(r_1, \alpha) = p_0 \sum_{n=1,2,3,\ldots}^{\infty} \frac{(-1)^n r_1^{-(2n+1)} \cos[(2n+1)(\alpha - \alpha^+)]}{\pi(n+1)(2n+1)}, \quad r_1 > 1.$$

If we assume that the unloading of the medium is rigid, i.e. occurs without any change in the previously acquired density, then the pressure behind the front of the loading wave in the medium has the form

$$p(r_1, \alpha) = p_0\left\{1 + \frac{r_1 \sin(\alpha + 2\beta)}{\sin 2\beta} + \right.$$
$$\left. + \sum_{n=1,2,3,\ldots}^{\infty} \frac{(-1)^n r_1^{\varkappa_n} \sin[\varkappa_n(\alpha - \alpha^+) + \beta]}{\varkappa_n(\pi n - 2\beta)}\right\}, \quad r_1 \leqslant 1, \qquad (1.4)$$
$$p(r_1, \alpha) = p_0\left\{-\frac{\sin\alpha}{2r_1\beta} + \sum_{n=1,2,3,\ldots}^{\infty} \frac{(-1)^n r_1^{-\gamma_n} \sin[\gamma_n(\alpha - \alpha^+) - \beta]}{\gamma_n(\pi n + 2\beta)}\right\}, \quad r_1 > 1,$$

where $\varkappa_n = 1 - \pi n/\beta$; $\gamma_n = 1 + \pi n/\beta$. Other possible cases are $D > a_1$ or $a_1 = \infty$, $\beta = \pi/2$ and are examined in a similar manner.

At the initial moment of time, the front of the loading wave contacts the shell at the point $\theta = 0$ (see Fig. 1). The wave front advances over time and the angle through which the shell encounters the wave increases. We will designate this angle as $\pm\alpha_*(t)$. Within the range $-\alpha_* \leq \theta \leq \alpha_*$ the shell is subjected to the action of a wave load whose front moves in the circumferential direction. We will henceforth describe pressure through the use of polar coordinate system ρ_1, θ, which is connected with the center of the shell. By means of parallel displacement and rotation of the coordinate axes y, z, we construct the system y_2, z_2 and, then, polar system ρ_1, θ. Meanwhile, we also take into account the change in the scale along the r axis, which was used to derive Eqs. (1.2-1.4). Coordinate systems r_1, α and ρ_1, θ are connected by the relations

$$\alpha = \mathrm{arctg}\,(\Delta_1/\Delta_2),\ r_1 = \Delta_1 \sin\alpha + \Delta_2 \cos\alpha. \tag{1.5}$$

Here, $\Delta_1 = Hk - \rho_1 \cos(\theta - \beta)$; $\Delta_2 = \rho_1 \sin(\theta - \beta) - \frac{1}{\sin\beta}(H\cos\beta - \rho_1) - t$; $H = H_0/l$; $\rho_1 = \rho_*/l$; $t = Dt_0/l$.

The following relation exists between the coordinate of the front of the wave load and time during the contact period

$$\alpha_*(t) = \arccos\left(1 - \frac{t\sin\beta}{R}\right),\quad t\sin\beta < R,\quad \alpha_*(t) = \pi/2,\ t\sin\beta > R,\ R = R_0/l.$$

The dimensionless time of contact of half the shell by the load $t_+ = R/\sin\beta$.

Numerical Example. A half-space was filled with sandy soil having the density $\rho = 1.35 \cdot 10^3$ kg/m³. The compression diagram of the soil is described by the power relation $p = p^0\varepsilon^n$, where $p^0 = 372$ MPa, ε is the volume strain, and the exponent $n = 3$ [2]. In determining the pressure (1.2-1.4), we approximate the diagram by means of linear functions with the bulk elastic modulus $K_{1d} = 11.3$ MPa, $K_{ud} = 180$ MPa. The value of K_{1d} corresponds to the velocity of the elastic wave in the given soil $a = 91$ m/sec, which was obtained experimentally in [2]. Taking the Poisson's ratio of the soil as $\nu_0 = 0.25$, we find the Lamé constants of the medium $\lambda = \mu = 6.75$ MPa. The parameters of the loading wave: $P_0 = 2$ MPa, $\ell = 2$ m, $D = 200$ m/sec. The shell was located at the depth $H_0 = 1$ m. The radius of the shell $R_0 = 0.5$ m, $h = 0.01$ m.

Figure 2 shows the diagram of pressure in the soil in relation to the coordinates r_1 and α. It can be seen that within the range $0 \leq r \leq 1$ the pressure at the wave front suddenly increases. It then decreases with an increase in the angle α and at $\alpha = \pi$ takes the value determined by Eq. (1.1). Pressure is low at $r_1 > 1.2$. It is positive within a narrow zone near $\alpha \sim \alpha^+$ of the front. With an increase in α, it changes sign, and it becomes zero at the boundary $z = 0$. The change in pressure over the radius within the range $0 \leq r \leq 1$ is close to linear.

Underground installations turn out to be loaded by a live load in the case of the action of pressure (1.2-1.4). Figure 3 shows a graph of pressure on a circular cylindrical surface of radius $\rho_* = 0.2275$ m in relation to the angle θ at $t = 0.05, 0.15, 0.30, 0.40$ (lines 1-4). The arrows show the direction of motion of the front of the load at the given moments of time. It can be seen that the pressure in the region $-\alpha_* \leqslant \theta \leqslant \alpha_*$ is initiallly close to uniform. Pressure becomes nonuniform as the angle of contact of the shell increases over time and reaches $\pm\pi/2$ at the moment $t_+ = 0.25\alpha_*$.

2. The authors of [3, 4] examined the reflection of a plastic plane shock wave from a flat barrier with normal incidence and incidence at an angle. In determining the wave load in the present case, we assume the shell to be rigidly fixed. We ignore diffraction phenomena and employ the principle of an isolated element. Then, using the results in [3, 4], we write the expression for the local reflection coefficient in the form

$$K_0 = K_* \cos\theta, \tag{2.1}$$

where $K_* = 1 + \sqrt{n}$ is the reflection coefficient with normal incidence of the wave front; θ is the angle of incidence at the moment of reflection. The pressure on the shell changes after reflection, since there is also a change in pressure (1.2-1.4) in the region $-\pi/2 \leqslant \theta \leqslant \pi/2$ in the neighborhood of the radius R_0.

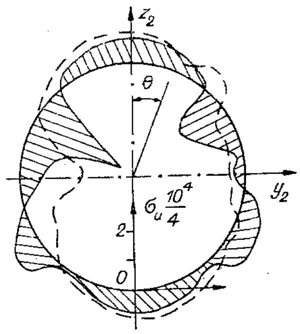

Fig. 4

In the case of displacement of the shell as a rigid cylinder or deformation, the surrounding medium offers resistance to this movement. The resistance force of the medium is taken to be proportional to the radial displacement of the shell and equal to $L_* w_0$ (where L_* is the proportionality factor and w_0 is the radial displacement of the shell).

We use the dimensionless quantities

$$w = w_0/R_0, \quad v = v_0/R_0, \quad p = P/\rho D^2, \quad b^2 = h^2/(12R_0^2),$$

and we write the equations of motion of the shell in the form [5]

$$\frac{\partial^2 v}{\partial\theta^2} + \frac{\partial w}{\partial\theta} = 0, \quad \frac{\partial v}{\partial\theta} + b^2\left(\frac{\partial^4 w}{\partial\theta^4} + 2\frac{\partial^2 w}{\partial\theta^2} + w\right) + (1 + q_*)w + \beta_1\frac{\partial^2 w}{\partial t^2} = -\beta_2 p_*(\theta, t). \tag{2.2}$$

Here, $\beta_1 = (1-\nu^2)\rho_0 D^2 R_0^2/(El^2)$; $\beta_2 = (1-\nu^2)\rho D^2 R_0 K_*/(Eh)$; $q_* = (1-\nu^2)R_0^2 L_*/(Eh)$; $p_*(\theta, t) = p(\theta, t)\cos\theta$; $p(\theta, t)$ is the pressure determined from Eqs. (1.2-1.5); meanwhile, $-\alpha_*(t) \leqslant \theta \leqslant \alpha_*(t)$; w_0, v_0 — are the radial and circumferential components of the displacement of the shell; ρ_0, ν, and E are the density, Poisson's ratio, and elastic modulus of the shell material. For the displacements, we take the initial conditions

$$w(\theta, t) = \frac{\partial}{\partial t}w(\theta, t) = v(\theta, t) = \frac{\partial}{\partial t}v(\theta, t) = 0 \quad (t = 0).$$

The pressure on the shell is asymmetric relative to the axis $\theta = 0$, and we seek the deflections in the form of the expansion

$$w = \sum_{m=0}^{\infty}[W_{1m}(t)\cos m\theta + W_{2m}(t)\sin m\theta],$$
$$v = \sum_{m=1}^{\infty}[V_{1m}(t)\sin m\theta + V_{2m}(t)\cos m\theta]. \tag{2.3}$$

We expand the external load into the series

$$p_*(\theta, t) = \frac{a_0}{2} + \sum_{m=1}^{\infty}(a_m\cos m\theta + b_m\sin m\theta), \tag{2.4}$$

where $a_m = \frac{1}{\pi}\int\limits_{-\alpha_*(t)}^{+\alpha_*(t)} p(\theta, t)\cos\theta\cos m\theta\, d\theta$ $(m=0,1,2,3,\ldots)$; $b_m = \frac{1}{\pi}\int\limits_{-\alpha_*(t)}^{+\alpha_*(t)} p(\theta, t)\cos\theta\sin m\theta\, d\theta$ $(m=1,2,3,\ldots)$.

An expression for the proportionality factor L_* was obtained in [6] by simultaneously solving the equations of motion of the shell and the surrounding elastic medium with the action of a harmonic compression wave. This is a complex expression which depends on the physical and geometric parameters of the medium and the shell, as well as the mode of deformation. Here, we use the static variant of these coefficients

$$L_* = L_0 = 2(\lambda + 2\mu)/R_0, \quad L_* = L_m = 4\mu(m+1)/(R_0 m), \tag{2.5}$$

which correspond to axisymmetric motion and motion of the shell with the formation of m diametrical nodal lines.

Inserting Eqs. (2.3-2.5) into equations of motion (2.2) and equating coefficients with sin $m\theta$ and cos $m\theta$ to zero for each m, we obtain the system

$$\begin{aligned} &\beta_1 \ddot{W}_0 + \omega_0^2 W_0 = -\beta_2 \frac{a_0}{2}, \quad m = 0, \\ &\left.\begin{aligned} &\beta_1 \ddot{W}_{1m} + \omega_m^2 W_{1m} = -\beta_2 a_m, \quad V_{1m} = -\frac{1}{m} W_{1m} \\ &\beta_1 \ddot{W}_{2m} + \omega_m^2 W_{2m} = -\beta_2 b_m, \quad V_{2m} = \frac{1}{m} W_{2m} \end{aligned}\right\} m \geqslant 1, \end{aligned} \tag{2.6}$$

where $\omega_m^2 = b^2(m^2-1)^2 + q_m$; $\omega_0^2 = 1 + b^2 + q_0$; $q_m = (1-\nu^2)R_0^2 L_m/(Eh)$; $q_0 = (1-\nu^2)R_0^2 L_0/(Eh)$; the dots denote differentiation with respect to time.

We solve Eqs. (2.6) by the method of variation of arbitrary constants

$$\begin{aligned} W_0(t) &= -\frac{\beta_2}{2\beta_1\omega_0^*}\int_0^t a_0(\tau)\sin\omega_0^*(t-\tau)\,d\tau, \\ W_{1m}(t) &= -\frac{\beta_2}{\beta_1\omega_m^*}\int_0^t a_m(\alpha_*, \tau)\sin\omega_m^*(t-\tau)\,d\tau, \\ W_{2m} &= -\frac{\beta_2}{\beta_1\omega_m^*}\int_0^t b_m(\alpha_*, \tau)\sin\omega_m^*(t-\tau)\,d\tau. \end{aligned} \tag{2.7}$$

Here, $\omega_0^* = \omega_0/\sqrt{\beta_1}$; $\omega_m^* = \omega/\sqrt{\beta}$. The quantity W_1 corresponds to the motion of the shell as a rigid cylinder.

We use the solutions of (2.7) and the formula $M = \frac{Eh^3}{12(1-\nu^2)R_0^2}\left(\frac{\partial^2 w_0}{\partial\theta^2} - \frac{\partial v_0}{\partial\theta}\right)$, to find the stresses in the shell. After performing certain transformations, we reduce the expressions for the dimensionless displacement and the bending stress to the form

$$\begin{aligned} w(\theta, t) = W_0(t) - \frac{\Lambda}{h}\sum_{m=2}^{\infty} F_m(\theta, t), \quad \sigma_u(\theta, t) = \\ = -\frac{\Lambda}{2R_0}\sum_{m=2}^{\infty}(m^2-1)F_m(\theta, t), \end{aligned} \tag{2.8}$$

where $F_m(\theta, t) = \frac{1}{\omega_m^*}\int_0^{t}\int_{-\alpha_*}^{+\alpha_*} p(x, \tau)\cos x \cos m(x-\theta)\sin\omega_m^*(t-\tau)\,dx\,d\tau$; $\Lambda = l^2\rho K_*/(\pi R_0\rho_0)$. The dimensional and dimensionless stresses are connected by the relation $\sigma_u = \sigma_{u*}(1-\nu^2)/E$.

Figure 4 shows a graph of the stresses in the shell in relation to θ at the moments of time t = 0.2 and 0.3 (solid and dashed lines). All of the calculations were performed on an "Elektronika-D3-28" computer. In summing the series (1.2-1.4) and (2.8), the number of terms of the series in the partial sum Σ_n was determined each time from the condition $S_{n+1}/\Sigma_n \leq 0.05$, where S_{n+1} is the (n + 1)-th term of the series. We used (2.8) to determine the stresses at fixed points of the shell cross section as a function of time at intervals of 0.05. About 2 min were required for the computation at the given point. Data from this calculation is shown in Fig. 4.

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THEORY OF INELASTIC STRAIN BASED ON NONEQUILIBRIUM OF THE STATE OF THE MATERIAL

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As any process which occurs at a finite rate, the inelastic strain of solids is always a thermodynamically nonequilibrium process. The transition from the given state to the equilibrium state is completed by stress relaxation, which converts elastic strain to inelastic strain. The theory of the deformation of solids has been constructed within this framework.

The basis of the classical theory of plasticity – the formation of inelastic strain when the given process is occurring in the equilibrium state – is only a convenient hypothesis [1, 2]. It leads to results which agree only with those experiments in which the rate of change in the external parameters is not greater than the rate of transition of the system (specimen) from the nonequilibrium state to the thermodynamically equilibrium state. In the theory of plasticity, such processes are referred to as quasistatic processes. In fact, fixing the external parameters in these processes means simultaneously fixing the parameters throughout the system as a whole. Indeed, nonequilibrium also exists during the process of plastic deformation, but the transition from this to the equilibrium state occurs only with a change in the external parameters – and is not seen after the latter become fixed. If the rate of change in these parameters is greater than the rate of transition of the system from the nonequilibrium state to the equilibrium state, then the nonequilibrium remains even after the external parameters stop changing. Thus, if they are fixed and subsequently kept constant, then the transition from the nonequilibrium state to the equilibrium state and associated phenomena, such as the formation of plastic strain, will continue until the establishment of thermodynamic equilibrium. The study of these phenomena was taken up in [3, 4].

If nonequilibrium during plastic deformation is not taken into account (and the state is assumed to be an equilibrium state), then, in accordance with the principle of the existence of a ground state, such deformation is a reversible process [5-7], i.e. the laws of thermodynamics are violated. Thus, the process cannot occur in nature. It follows from this that the theory of plasticity is only a model representation of the phenomenon of plastic inelastic deformation. It describes it as a process which occurs under certain conditions. Neither the theoretical or empirical accuracy requirements are high and the the existence of nonequilibrium is ignored. In fact, any inelastic deformation, including plastic deformation, is a nonequilibrium process. It is on this basis that the theory of plastic deformation has been constructed.

1. Equilibrium and Nonequilibrium Stresses. Proceeding as in the nonequilibrium thermodynamics of solids and basing our theory on the main conservation laws and the principles of objectivity, continuity, locality, and the existence of a ground state, we make the transition from an actual solid to a continuum and, within this continuum, we make

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